Logarithms Workshop

Inverses:

Definition of an Exponential Function:
An Exponential function is a function that can be written in the form
$f(x) = b^x$
Where b is a positive constant, $b \neq 1$.

Definition of a logarithm:
$y = \log_b x$ if and only if $x = b^y$, Where b is a positive constant, $b \neq 1$.

Common Log:
Has a base b=10, is notated as log(x). In other words, $\log(x) = \log_{10}(x)$.

Natural Log:
Has a base of b=e, where $e \cong 2.718$, is notated as ln(x). In other words, $\ln(x) = \log_e(x)$.

Properties of Logarithms:
$\log_b(b^x) = x$
$b^{\log_b(x)} = x$

$$\log_b(x \cdot y) = \log_b x + \log_b y$$
$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$
$$\log_b x^y = y \cdot \log_b x$$

Using Logs to Solve Equations:

Example: Solve the following equation for x.

$$10^{2x} \cdot 1000 = .001^{x+1}$$

Logs in Chemistry:

pH is defined as the negative logarithm of the hydrogen ion concentration. The equation is:

$pH = -\log [H^+]$

or equivalently, when solved for the hydrogen ion concentration, we have

$[H^+] = 10^{-pH}$

Chemistry Example1: If an acid has an H^+ concentration of 0.00001 M, find the pH.

Solution:

First convert the number to exponential notation, find the log, then solve the pH equation.

$H^+ = 0.00001M = 10^{-5}$

$\log(H^+) = \log(10^{-5})$

$\log(H^+) = -5$

$-\log(H^+) = 5$

$pH = 5$

The purpose of the negative sign in the log definition is to give a positive pH value.

Chemistry Example2: If an acid has an H^+ concentration of 0.0023 M, find the pH.

Solution:.

$[H^+]$= 0. 0023 M
$\log[H+] = \log(.0023)$
$\log[H+] = -2.638$ (note that you need a calculator to determine that log(.0023)=-2.638
$-\log[H+] = 2.638$
$pH = 2.638$

Chemistry Example3: If a solution has a pH = 3.5, find the hydrogen ion concentration $[H^+]$

Solution – Method 1:	
$pH = 3.5$	
$-\log[H+] = 3.5$	Using the pH equation
$\log[H+] = -3.5$	
$10^{\log[H+]} = 10^{-3.5}$	make each side of the equation the exponent for 10
$[H+]=10^{-3.5}$ M	
$[H+] = .0003162$ M	using a calculator
$[H+] = 3.162 \times 10^{-4}$ M	Put in scientific notation
Solution – Method 2:	
$[H^+] = 10^{-pH}$	Using the alternative hydrogen ion concentration equation,
$[H^+] = 10^{-3.5}$	Plug in the pH
$[H^+] = 3.162 \times 10^{-4}$ M	Write the number in the scientific notation.